PROBLEM: A doctor finds evidence of a serious illness in a particular patient and must make a determination about whether or not to advise the patient to undergo a dangerous operation. If the patient does suffer from the illness in question, there is a 95% probability that he will die if he does not undergo the operation. If he does undergo the operation, he has a 50% probability of survival. If the operation is conducted and it is discovered that the patient does not suffer from the illness, there is a 10% probability that the patient will die due to complications resulting from the operation. If it has been estimated that there is a 20% to 30% probability of the patient actually having the illness in question, how should the doctor advise her patient?

ATTEMPTED SOLUTION:
We will use a chance tree to solve this problem.

patient	has illness	surgery	dies	0.2*0.5=0.1
			survives	0.2*0.5=0.1
		no surgery	dies	0.2*0.95 = 0.19
			survives	0.2*0.05=0.01
	has no illness	surgery	dies	0.8*0.1=0.08
			survives	0.8*0.9=0.72
		no surgery	dies	0
			survives	0.8*1=0.8

Now let A be the event that this individual dies with surgery, and let B be the event that the person actually has the disease. The individual is interested in P(A|B). Then

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.2} = 0.5$$

Let A' be the event that this individual dies without surgery, and let B be the event that the person actually has the disease. The individual is interested in P(A'|B). Then

$$P(A' \mid B) = \frac{P(A' \cap B)}{P(B)} = \frac{0.19}{0.2} = 0.95$$

From the above calculation it is clear that the doctor should advise the patient to take the surgery.

PROBLEMS WITH THIS SOLUTION

1. the probabilities in the chance tree don't add up to 1
2. can we define the event B as we did, since we only know the patient has been diagnosed with the disease, not that he certainly has it.